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METEOROID DAMAGE TO FILAMENTARY STRUCTURES

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By Richard H. MacNeal
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SUMMARY

Fracture rates due to micrometeoroid impact are computed for solid round wires, flat tapes and thin-walled hollow tubes. The calculations are based on an extension of test data for the penetration of flat plates and on recent (1965) estimates of meteoroid flux. The results show that solid wires are much more vulnerable to meteoroid damage than either flat tapes or hollow tubes.

INTRODUCTION

A promising area of present research and future application is the use of slender filamentary structures in space missions. Such structures are especially advantageous when it is required to span a large distance or to cover a large area as, for instance, in the case of an orbiting radio telescope.

The tension members in large structures used in space will tend to be extremely fine (of the order of a few mils in diameter) because the mechanical loads in a space environment are extremely small. The combination of small loads, long lengths, and small cross sections increases the probability that fracture due to meteoroids will be the critical design consideration for tension members. Compression members, on the other hand, will in most instances be designed by elastic stability.

Structural designs employing filamentary structures in space clearly require a means for estimating fracture rates due to micrometeoroids. The technical literature is at present (1966) devoid of reference to experimental or theoretical studies specifically directed to the topic. The study reported here is an attempt to fill the present need by means of a plausible extension of published work on related subjects such as the cratering of semi-infinite solids and the penetration of plates by high velocity impacts. No new experimental results or mathematical theories of fracture are presented.

The cross-sectional shape of a filament has an important effect on the selection of the approach used in estimating its fracture rate. The report is, accordingly, organized into separate sections on solid wires, flat tapes and hollow tubes. The results indicate, incidentally, large differences in the vulnerabilities of these three shapes.

It has been necessary to make several arbitrary or merely plausible assumptions in the analysis, particularly in regard to numerical values of empirical coefficients. It is believed, however, that the uncertainties introduced thereby into the results are less than the uncertainties due to the present incomplete knowledge of the meteoroid environment.

SYMBOLS

A	cross-sectional area
D	hole diameter in thin plate
d_c	crater depth
d_p	particle diameter
d_t	diameter of hollow tube
d_w	wire diameter
F_{tu}	ultimate strength of target material
k	number of redundant parallel elements
ℓ	length of member
m_p	particle mass
N_f	fracture rate, fractures/unit length/unit time
N_p	cumulative meteoroid flux, particles/unit area/unit time

P_e	probability that a single element of a member with redundant elements has failed
P_m	probability that a single member has failed
t	time
t_p	plate thickness
t_s	thickness of flat tape
t_t	wall thickness for hollow tube
v_o	empirical velocity parameter (6.5 km/sec)
v_p	particle velocity
w_s	width of flat tape
η	proportion of failed members
θ	angle of particle incidence, measured from normal to surface
θ_o	incidence angle above which minimum particle size required to produce fracture is assumed to remain constant
ρ_p	particle density

THE MICROMETEOROID ENVIRONMENT IN NEAR-EARTH ORBIT

Cumulative micrometeoroid flux is defined as the number of particles greater than a certain size passing through a unit surface area from all directions in a unit time. The dependence of cumulative flux on particle size is usually expressed in the form

$$N_p = 10^a \cdot m_p^{-b} \tag{1}$$

or

$$\log_{10} N_p = a - b \cdot \log_{10} m_p \tag{2}$$

The unit of mass most commonly used is the gram and the unit of N most commonly used is particles/meter^2/sec. Estimates from several sources of micrometeoroid flux in Earth orbit are plotted in Figure 1.

The curve labeled "Gen. Motors 1965" is the mean of "optimistic" and "pessimistic" estimates published in Reference 5, which are, respectively, one order of magnitude lower and higher. Recent experimental results involving penetration of panels from Explorers XVI, XXIII, Pegasus, and Mariner IV confirm the accuracy of the General Motors estimate which shall be used in the present study. The mathematical description of the curve is

$$N_p = \frac{10^{-14}}{m_p} \text{ particles/meter}^2\text{/sec} \tag{3}$$

where m_p is in grams.

The curve labeled "NASA Houston 1965" is in reality a slight modification of the Whipple 1963 curve and therefore does not reflect the most recent results from spacecraft experiments. It is probably unduly pessimistic.

The geocentric velocity of primary particle flux can vary between 11 km/sec and 72 km/sec. The average velocity is frequently assumed to be 30 km/sec. The velocity of the spacecraft (7 km/sec) should be added vectorially to this value, but it produces little change in the result. 30 km/sec shall be adopted in the present study as the average velocity of particles relative to the spacecraft.

Estimates of average micrometeoroid density range from .05 to about 8.0 grams/cm^3. The average density will, for the present study, be taken as 0.5 grams/cm^3. This is the value recommended in Reference 3.

FRACTURE OF ROUND WIRES BY MICROMETEOROIDS

No direct experimental data is available from which the momentum (or kinetic energy) of a hypervelocity particle required to fracture a wire can be calculated. An estimate will be made that is based on extrapolation of puncture data for flat plates.

It has been reported by many investigators (Refs. 5 and 7) that penetration of flat plates due to the normal impact of hypervelocity particles results if the crater depth computed for a semi-infinite target exceeds approximately two-thirds of the plate thickness. We are, therefore, led to a consideration of impacts on semi-infinite solids.

Experimental results for the depth of craters in semi-infinite solids due to impact by hypervelocity particles are surveyed in Reference 6, where it is proposed that the crater depth be computed by the semi-empirical formula

$$d_c = \left(\frac{m_p V_p^2}{4\pi S}\right)^{1/3} \tag{4}$$

where

m_p = mass of particle

V_p = particle velocity

S = characteristic material stress of target, either ultimate tensile strength or true tensile stress to fracture

If Equation (4) is amended to read

$$d_c = \left(\frac{m_p V_p V_o}{4\pi F_{tu}}\right)^{1/3} \tag{5}$$

where V_o = 6.5 km/sec and F_{tu} is the ultimate tensile stress, a better fit is obtained to the data reported in Reference 6. A comparison between measured and calculated crater depths is plotted in Figure 2. It will be noted that the crater depth is proportional to the cube root of particle kinetic energy in Equation (4), and to the cube root of particle momentum in Equation (5). Most mathematical models of cratering lead to one or the other formulation. See Reference 5 for discussion.

There is danger in the use of any currently available formula for meteoroid penetration due to the rather large extrapolation required with respect to velocity (9 km/sec $\longrightarrow$ 30 km/sec). Although the agreement between Equation (5) and experiment is good, penetration depth will be assumed to depend on the square-root of velocity, in deference to the 2/3 power law preferred by most theoreticians. The following formula is, therefore, used in the present report

$$d_c = \left(\frac{m_p V_p^{3/2} V_o^{1/2}}{4\pi F_{tu}} \right)^{1/3} \tag{6}$$

where V_o = 6.5 km/sec. A physical basis for V_o is provided by noting that it is approximately equal to the velocity of sound for common metals.

Crater depth may be related to particle size by assuming that the hypervelocity particles are spheres so that

$$m_p = \frac{\pi}{6} \cdot \rho_p d_p^3 \tag{7}$$

where

ρ_p = particle density

d_p = particle diameter

Substituting into Equation (5)

$$\frac{d_c}{d_p} = \left(\frac{\rho_p V_p^{3/2} V_o^{1/2}}{24 F_{tu}} \right)^{1/3} \tag{8}$$

As an example let

ρ_p = .5 gm/cm^3 (estimated mean density of meteoroids)

V_p = 3.0×10^6 cm/sec (estimated mean velocity of meteoroids)

V_o = $.65 \times 10^6$ cm/sec

F_{tu} = 3×10^9 dyne/cm^2 (42 500 psi) (similar to Aℓ-6061-T6)

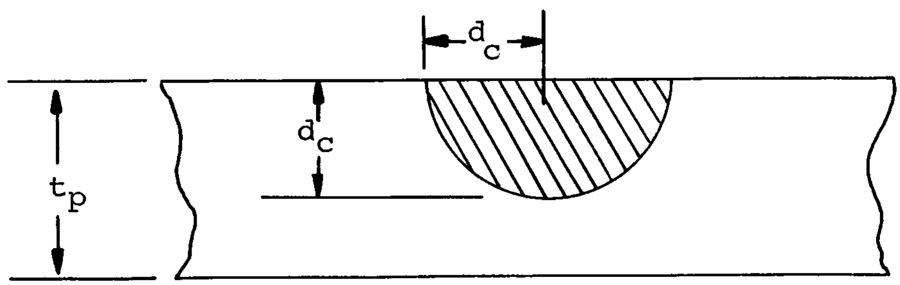
Then

$$\frac{d_c}{d_p} = \left(\frac{.5 \times (3.0 \times 10^6)^{3/2} \times (.65 \times 10^6)^{1/2}}{24 \times 3 \times 10^9} \right)^{1/3} = (29.1)^{1/3}$$

$$= 3.06$$

This result will, in certain calculations, be taken as the typical value for meteoroid impact.

The fracture of a round wire may be related to the penetration of a flat plate by comparing the cross-sectional area that is available to carry transverse shear. In the case of a flat plate (shown below) the shear area is equal to $2\pi t_p \cdot d_c$ assuming the crater to be a hemisphere.



The shear resisting area of a round wire of diameter, d_w, is equal to $2\pi\left(\frac{d_w}{2}\right)^2$, assuming the effective surfaces to be those normal to the axis of the wire. Equating shear areas, the wire diameter for fracture is

$$d_w = 2\sqrt{t_p d_c} \tag{9}$$

Plate thickness for penetration may be related to crater depth in a semi-infinite target. Careful experiments, Reference 5, indicate that penetration depends on material properties and that the "equivalent" crater depth is between .5 and .7 times the plate thickness. For convenience, assume that $t_p = 1.56\ d_c$ is the plate thickness for penetration. Then the wire diameter for fracture is

$$d_w = 2\sqrt{1.56}\cdot d_c = 2.5\cdot d_c \tag{10}$$

so that

$$d_c = 0.4\cdot d_w \tag{11}$$

is the "equivalent" crater depth that will just break the wire. Using Equation (6) for crater depth, the relationship between wire diameter, material properties, and particle kinetics for wire fracture is

$$d_w \leq 2.5\left(\frac{m_p V_p^{3/2} V_o^{1/2}}{4\pi F_{tu}}\right)^{1/3} \tag{12}$$

It will be assumed that any particle whose center of gravity intersects the wire, even at a glancing angle, will break the wire provided that Equation (12) is satisfied.

We are now in a position to estimate the fracture rate of round wires. The particle mass that will just break the wire is, from Equation (12)

$$m_p = \frac{4\pi F_{tu}}{V_p^{3/2} V_o^{1/2}}\cdot\left(\frac{d_w}{2.5}\right)^3 \tag{13}$$

The number of fractures per unit length of wires per unit time is related to the meteoroid flux of particles with mass greater than m_p by

$$N_f = \frac{d_w}{100}\cdot\frac{\pi}{2}\cdot N_p \quad \text{fractures/meter/sec} \tag{14}$$

where d_w is measured in centimeters.

The factor $\frac{\pi}{2}$ is introduced to account for the fact that a round wire has $\pi/2$ times as much surface area as a flat strip of equal dimension and is, therefore, somewhat more susceptible to collision with small particles.

Using Equation (3) for particle flux

$$N_f = \frac{\pi}{2}\left(\frac{d_w}{100}\right)\frac{10^{-14}}{m_p} = \frac{\pi}{2}\cdot d_w\cdot\frac{10^{-16}V_p^{3/2}V_o^{1/2}}{4\pi F_{tu}} \times \left(\frac{2.5}{d_w}\right)^3$$

$$= \frac{1.95 \times 10^{-16}}{\left(d_w\right)^2}\cdot\frac{V_p^{3/2}V_o^{1/2}}{F_{tu}} \quad \text{fractures/meter/sec} \qquad (15)$$

As an example let

V_p = 3.0×10^6 cm/sec (average meteroid velocity)

V_o = $.65 \times 10^6$ cm/sec (empirical factor)

F_{tu} = 3×10^9 dyne/cm^2 = 42 500 psi, (Aℓ-6061-T6)

Then

$$\frac{V_p^{3/2}V_o}{F_{tu}} = \frac{(3.0)^{3/2} \times (.65)^{1/2} \times 10^{12}}{3 \times 10^9} = 1400$$

and

$$N_f = \frac{1.95 \times .14 \times 10^{-12}}{d_w^2} = \frac{.273 \times 10^{-12}}{d_w^2} \quad \text{fractures/meter/sec}$$

It is perhaps more meaningful to express N_f in fractures/meter/year and d_w in mils (.001").

Since

$$1 \text{ year} = .316 \times 10^8 \text{ sec}$$

and

$$1 \text{ mil} = .00254 \text{ cm}$$

$$N_f = \frac{.273 \times 10^{-12} \times .316 \times 10^{8}}{(.00254)^2} \times \frac{1}{\left(d_w\right)^2_{mils}}$$

$$= \frac{1.328}{\left(d_w\right)^2_{mils}} \quad \text{fractures/meter/year}$$

It is evident that a one-mil aluminum wire is not suitable for long term use in space. Equation (15) is plotted in Figure 3 for various material strengths.

FRACTURE OF FLAT TAPES BY MICROMETEOROIDS

We have assumed that a round wire will be fractured by a particle if the crater formed by the particle in a semi-infinite target has a depth equal to .40 times the wire diameter. Also, for micrometeoroids impacting on 6061-T6 aluminum, the particle diameter has been shown to be equal to about one-third the crater depth. Thus, meteoroids can be expected to break round aluminum wires if their diameter exceeds .133 times the wire diameter.

In the case of a thin flat sheet on the other hand, we expect that a particle that is large compared to the thickness of the sheet will produce a hole in the sheet that is scarcely larger than the diameter of the particle. This expectation has been confirmed by experimentation with hypervelocity particles (Reference 8) where it is shown that the following empirical formula agrees well with experimental results

$$\frac{D}{d_p} = .45\, V_p \left(\frac{t_s}{d_p}\right)^{2/3} + .90 \tag{16}$$

where
- D = diameter of hole
- d_p = diameter of particle
- t_s = sheet thickness
- V_p = particle velocity in kilometers/sec

For normal impact on a flat strip it will be assumed that the strip will be fractured by any particle whose center of gravity intersects the strip provided that the resulting hole diameter exceeds .75 times the width of the strip. Thus, a strip of width w_s will be fractured if

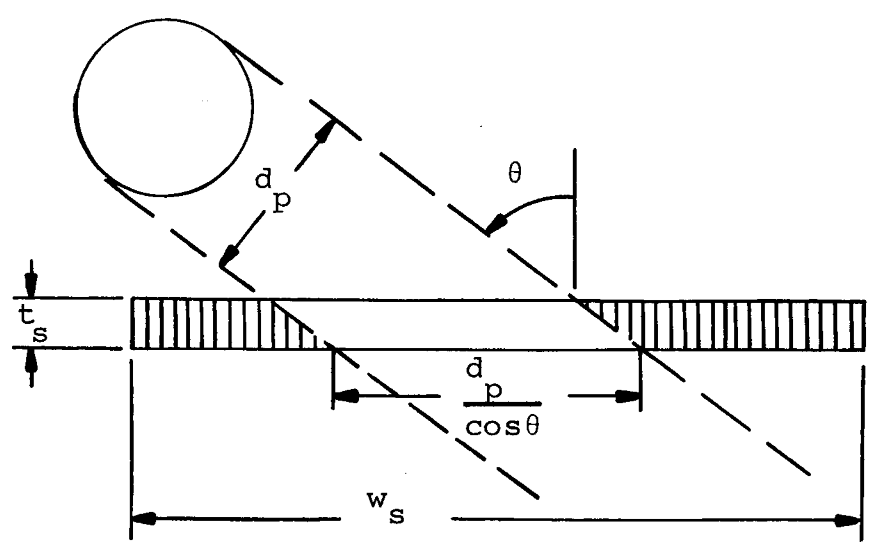
$$\frac{w_s}{d_p} \le \frac{D}{.75d_p} = \frac{1}{.75}\left[.45V_p\left(\frac{t_s}{d_p}\right)^{2/3} + .90\right]$$

or

$$\frac{w_s}{d_p} \le \frac{1}{.75}\left[.45V_p\left(\frac{w_s}{d_p}\right)^{2/3}\cdot\left(\frac{t_s}{w_s}\right)^{2/3} + .90\right] \qquad (17)$$

The relationship between ${}^{w}s/d_p$ and the thickness ratio of the strip, ${}^{t}s/w_s$, is plotted in Figure 4 for an assumed particle velocity equal to 30 km/sec. It is seen that the ${}^{w}s/d_p$ increases slowly with strip thickness. It will be assumed that ${}^{w}s/d_p = 1.5$ in subsequent analysis.

An inclined impact has a higher probability of fracture than a normal impact as is evident from the following diagram.



It will be assumed that, if the center of the particle intercepts the strip, the strip will be fractured provided that $\frac{d_p}{\cos\theta} > \frac{w_s}{1.5}$ except for very large incidence angles, as explained below.

For large incidence angles it will further be required, in order for fracture to occur, that the particle diameter be large enough to produce fracture of a round wire of equal cross-sectional area. This requirement is introduced in order to avoid the mathematical singularity that results if infinitesimally small particles can fracture tapes at grazing incidence. It is justified by the argument that conversion of kinetic energy into heat is a major factor in the fracture process.

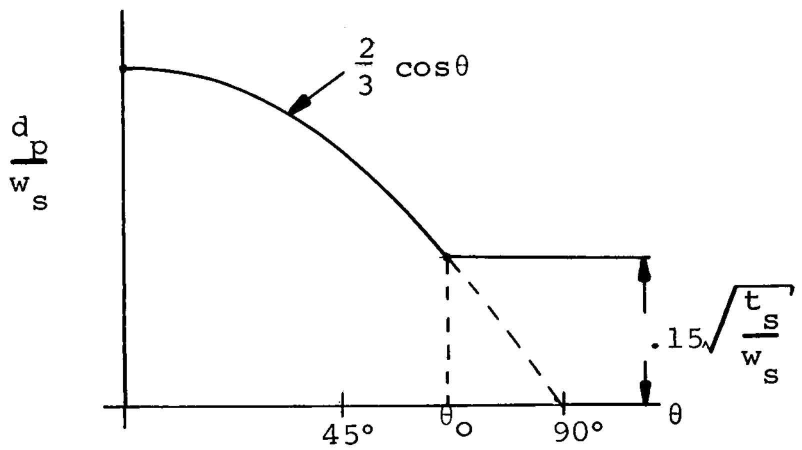
It has been estimated at the beginning of this section that a meteoroid will break a round aluminum wire if its diameter exceeds .133 times the wire diameter, i.e., if

$$d_p > .133\sqrt{\frac{4}{\pi}\cdot A_w} = .133\sqrt{\frac{4}{\pi}\cdot t_s w_s}$$

or

$$d_p > .15\, w_s \sqrt{\frac{t_s}{w_s}} \tag{18}$$

The relationship between minimum particle size and incidence angle is plotted below



For $\theta > \theta_o$, the minimum particle size to produce fracture remains constant, where

$$\frac{2}{3} \cdot \cos\theta_o = .15\sqrt{\frac{t_s}{w_s}} \tag{19}$$

In order to estimate fracture rate, we require the probability distribution of particle interception vs. incidence angle. Since particle flux is homogenous with respect to direction, the rate of interception for a given direction is proportional to the projection of the area of the strip on the particle direction. Thus the rate of interception of particles with incidence between θ and $\theta + \Delta\theta$ is

$$n_p(\theta + \Delta\theta) - n_p(\theta) = N_p \cdot \cos\theta \cdot \Delta\theta \tag{20}$$

where N_p is the total flux of particles from all directions.

The fracture rate per unit length, per unit time, for a strip of width w_s is

$$N_f = w_s\left[\int_o^{\theta} N_p\left(\frac{2w_s}{3} \cdot \cos\theta\right) \cos\theta \cdot d\theta + \int_{\theta_o}^{\pi/2} N_p\left(\frac{2w_s}{3} \cdot \cos\theta_o\right) \cos\theta \cdot d\theta\right] \tag{21}$$

where $N_p\left(\frac{2w_s}{3} \cdot \cos\theta\right)$ is the flux of particles with diameter greater than $\frac{2w_s}{3} \cdot \cos\theta$.

The particle flux as a function of particle mass is given by Equation (3). Using Equation (7) to relate particle mass to particle diameter

$$N_p\left(d_p\right) = 10^{-14} \cdot \frac{6}{\pi\rho_p d_p^3} \text{ particles/meter}^2\text{/sec} \tag{22}$$

where d_p is measured in centimeters and ρ_p in grams/cm^3 .

So that

$$N_p\left(\frac{2w_s\cdot\cos\theta}{3}\right) = \frac{6 \times 10^{-14}}{\pi\rho_p} \times \left(\frac{3}{2w_s\cdot\cos\theta}\right)^3 = \frac{6.45 \times 10^{-14}}{\rho_p w_s^3\cdot\cos^3\theta} \tag{23}$$

Substituting into Equation (21) and dividing by 100, so that w_s may be expressed in centimeters and N_f may be expressed in fractures/meter/sec,

$$N_f = \frac{6.45 \times 10^{-16}}{\rho_p w_s^2}\left[\int_0^{\theta_o} \frac{d\theta}{\cos^2\theta} + \int_{\theta_o}^{\pi/2} \frac{\cos\theta\cdot d\theta}{\cos^3\theta_o}\right] \tag{24}$$

$$= \frac{6.45 \times 10^{-16}}{\rho_p w_s^2}\left[\tan\theta_o + \frac{1}{\cos^3\theta_o}\left(1 - \sin\theta_o\right)\right] \tag{25}$$

Since θ_o is a little less than 90°

$$\tan\theta_o \simeq \frac{1}{\cos\theta_o}$$

$$1 - \sin\theta_o = 1 - \sqrt{1 - \cos^2\theta_o} \simeq \frac{1}{2}\cdot\cos^2\theta_o$$

Thus the particles with incidence angles greater than θ_o produce one-third of the fractures, and the total fracture rate is

$$N_f = \frac{6.45 \times 10^{-16}}{\rho_p w_s^2}\left[\frac{3}{2\cdot\cos\theta_o}\right]$$

and, using Equation (19),

$$N_f = \frac{4.3 \times 10^{-15}}{\rho_p w_s^2}\left(\frac{w_s}{t_s}\right)^{\frac{1}{2}} \quad \text{fractures/meter/sec} \tag{26}$$

Converting w_s to mils and the time period to years

$$N_f = \frac{4.3 \times 10^{-15} \times .316 \times 10^{8}}{(.00254)^2} \times \frac{1}{\rho_p w_s^2}\left(\frac{w_s}{t_s}\right)^{\frac{1}{2}}$$

$$= .021 \times \frac{1}{\rho_p w_s^2}\left(\frac{w_s}{t_s}\right)^{\frac{1}{2}} \tag{27}$$

Let $\rho_p = .5$ grams/cm^3 . Then

$$N_f = \frac{.042}{\left(w_s\right)^2}\left(\frac{w_s}{t_s}\right)^{\frac{1}{2}} \quad \text{fractures/meter/year} \tag{28}$$

Comparing this result with that for a round aluminum wire we find that

$$\frac{\left(N_f\right)\text{strip}}{\left(N_f\right)\text{wire}} = \frac{.042}{1.328}\left(\frac{d_w}{w_s}\right)^2 \cdot \left(\frac{w_s}{t_s}\right)^{\frac{1}{2}} \tag{29}$$

Let the cross-sectional area of the strip and of the wire be the same. Then

$$d_w^2 = \frac{4}{\pi} \cdot A$$

$$w_s^2 = A \cdot \frac{w_s}{t_s}$$

$$\frac{\left(N_f\right)\text{strip}}{\left(N_f\right)\text{wire}} = \frac{.042}{1.328} \times \frac{4}{\pi}\left(\frac{t_s}{w_s}\right)^{\frac{1}{2}} \tag{30}$$

$$= .0403\left(\frac{t_s}{w_s}\right)^{\frac{1}{2}} \tag{31}$$

For equal cross-sectional area, it is seen that the fracture rate for a flat strip is much less than the fracture rate for a round wire. Equation (28) is plotted in Figure 5 for various thickness ratios. This plot does not take into account the dependence of hole size on thickness ratio shown in Figure 4. The effect will be to bring the curves for

$$\frac{w_s}{t_s} = 10 \qquad \text{and} \qquad \frac{w_s}{t_s} = 100$$

slightly closer together.

FRACTURE OF HOLLOW TUBES BY MICROMETEOROIDS

It is evident from preceding analysis that the fracture rate of flat tapes is increased by the vulnerability to edgewise impact. This particular vulnerability is avoided in the case of a thin-walled cylinder. Fracture rate for a circular cylinder can be estimated by assuming, by analogy with the flat tape, that a particle whose center intersects the tube will fracture it if the external tube diameter is less than 1.5 times the particle diameter.

The number of fractures per meter per second is

$$N_f = \frac{d_t}{100} \cdot \frac{\pi}{2} \cdot N_p\left(\frac{2}{3} \cdot d_t\right) \tag{32}$$

where d_t is the diameter of the tube measured in centimeters and $N_p\left(\frac{2}{3} \cdot d_t\right)$ is the flux of particles with diameter greater than $\frac{2}{3} \cdot d_t$. The factor $\frac{\pi}{2}$ is introduced to account for the fact that a round wire has $\frac{\pi}{2}$ times as much surface area as a flat strip of equal dimension. Using Equation (22) for particle flux

$$N_f = \frac{d_t}{100} \cdot \frac{\pi}{2} \cdot \frac{6 \times 10^{-14}}{\pi \rho_p} \left(\frac{3}{2d_t}\right)^3$$

$$= \frac{1.01 \times 10^{-15}}{\rho_p d_t^2} \quad \text{fractures/meter/sec} \tag{33}$$

Converting d_t to mils and the time period to years

$$N_f = \frac{1.01 \times 10^{-15} \times .316 \times 10^8}{(.00254)^2} \times \frac{1}{\rho_p d_t^2}$$

$$= \frac{.00494}{\rho_p d_t^2} \text{ fractures/meter/year} \tag{34}$$

Comparing this result with that for a flat strip, we find that

$$\frac{\left(N_f\right) \text{tube}}{\left(N_f\right) \text{strip}} = .235 \left(\frac{w_s}{d_t}\right)^2 \cdot \left(\frac{t_s}{w_s}\right)^{\frac{1}{2}} \tag{35}$$

Let the cross-sectional area of the strip and of the tube be the same. Then

$$A = w_s t_s = \pi d_t \cdot t_t$$

so that

$$\left(\frac{w_s}{d_t}\right)^2 = \pi^2 \left(\frac{t_t}{t_s}\right)^2$$

and

$$\frac{\left(N_f\right) \text{tube}}{\left(N_f\right) \text{strip}} = .235 \pi^2 \left(\frac{t_t}{t_s}\right)^2 \cdot \left(\frac{t_s}{w_s}\right)^{\frac{1}{2}} \tag{36}$$

Assuming that the wall thicknesses for the tube and the tape are comparable, the tube shows a substantial advantage only if the aspect ratio for the strip, w_s/t_s, is quite large.

The ratio of the fracture rates for a thin-walled hollow tube and a solid round aluminum wire is, assuming $\rho_p = .5$ gm/cm^3,

$$\frac{\left(N_f\right) \text{tube}}{\left(N_f\right) \text{wire}} = .00745 \left(\frac{d_w}{d_t}\right)^2 \tag{37}$$

Let the cross-sectional area of the tube and of solid wire be the same, then

$$A = \pi d_t t_t = \frac{\pi}{4} \cdot d_w^2$$

so that

$$\left(\frac{d_w}{d_t}\right)^2 = 4\left(\frac{t_t}{d_t}\right)$$

and

$$\frac{\left(N_f\right)_{tube}}{\left(N_f\right)_{wire}} = .0298\left(\frac{t_t}{d_t}\right) \tag{38}$$

Since $t_t/d_t \ll 1$ the fracture rate for the hollow tube is much less than that for the solid wire.

Equation (34) is plotted in Figure 5 to provide a comparison between the flat strip and the hollow tube.

ESTIMATION OF USEFUL STRUCTURAL LIFE

Most network-type space structures will, in all probability, consist of a large number of members that are redundant to the extent that the structure continues to fulfill a useful purpose until a significant proportion of the members have failed. Estimates of fracture rates contained in preceding sections can be used to estimate useful structural life or, conversely, to select a structural design that will have a specified useful life.

Let η be the ratio of the number of members that have failed to the total number. If the total number of members is large, η is very nearly equal to the probability, P_m, that a single member has failed:

$$\eta = P_m \tag{39}$$

If the member consists of a number, k, of identical elements in parallel, arranged such that the probability of a particle fracturing more than one element is vanishingly small and such that the member continues to function until all elements have failed, then the probability of failure of the member is

$$P_m = \left(P_e\right)^k \tag{40}$$

where P_e is the probability of failure of an individual element.

An elementary probability analysis shows that

$$P_e = 1 - e^{-N_f \ell t} \tag{41}$$

where N_f is the fracture rate per unit length of wire

ℓ is the length of wire

t is time

Thus, combining Equations (39), (40) and (41)

$$\eta = \left(1 - e^{-N_f \ell t}\right)^k \tag{42}$$

This relationship is plotted in Figure 6. It is seen that redundancy $(k > 1)$ increases useful life by a large factor only if the allowable proportion of failed members is small.

If the product $N_f \ell t$ is small then equation (42) may be approximated by

$$\eta = \left(N_f \ell t\right)^k \tag{43}$$

The allowable fracture rate is, for $N_f \ell t \ll 1$,

$$N_f = \frac{1}{\ell t}\left(\eta\right)^{1/k} \tag{44}$$

As a practical application of this formula, suppose that $\eta = 0.1$, $k = 1$, $\ell = 1$ meter, and $t = 10$ years. Then

$$N_f = .01 \quad \text{fractures/meter/year} \tag{45}$$

is the allowable fracture rate. This fracture rate is achieved (from Fig. 3) by an 11.5 mil solid aluminum wire, or (from Fig. 5) by a flat strip 3.7 mils wide and .37 mil thick.

CONCLUSIONS AND RECOMMENDATIONS

Quantitative conclusions from the present study must be stated with reservation due to the uncertainties introduced by assumptions and by the incomplete knowledge of the meteoroid environment. There appears, however, to be little doubt concerning the relative vulnerability of solid round wires, flat tapes and hollow tubes. The calculations show, for example, that if a fracture rate equal to 10^{-3} fractures/meter/year is acceptable, the minimum required dimensions for the three types of filaments are approximately as follows:

solid round wires	35 mil diameter
flat tapes	20 mil width
thin-walled hollow tube	3.2 mil diameter

It may be concluded that solid wires will not be competitive with the other shapes for applications requiring filaments with small cross-sectional areas.

Experimental studies on the fracture of filaments by hypervelocity particles are required in order to improve the reliability of estimated fracture rates. The need for such studies will increase as the errors due to uncertain knowledge of the meteoroid environment are diminished, and as interest in the use of filamentary structures for space applications grows.

Experiments with hypervelocity particle accelerators, and recoverable orbital experiments are both recommended. The experiments with particle accelerators will establish the relationship between particle size and wire diameter for the fracture of solid wires, and will illuminate the important question of the vulnerability of flat tapes to grazing impact. The recoverable orbital experiments will answer questions regarding the influences of particle density and velocity, and will provide proof tests in the actual meteoroid environment.

Astro Research Corporation
Santa Barbara, California, September 19, 1966.

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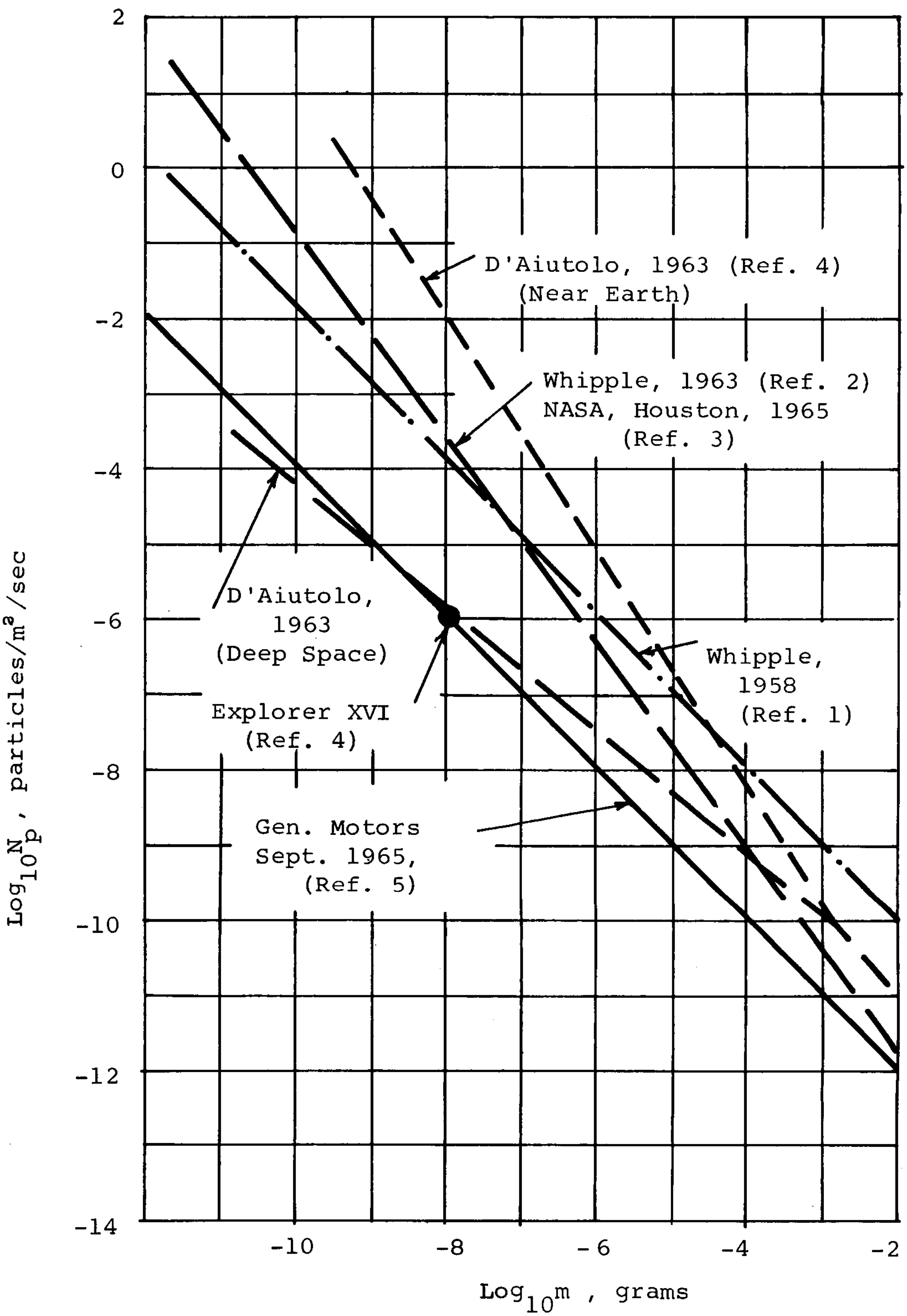

Figure 1. Estimates of Cumulative Micrometeoroid Flux

TARGETS	F_{tu}, dyne/cm^2 x 10^9
Al - 1100 - F	1.008
Al - 1100 - H14	1.243
Al - 2014 - T6	4.385
Al - 2017 - T4	4.447
Al - 6061 - T6	2.972
Al - 7075 - 0	2.372
Al - 7075 - T6	5.074
Cu - OFHC	6.660
Al - BS1476 - EICM	2.972

$$\frac{(d_c)_{measured}}{(d_c)_{computed}}$$

V_o = 6.5 km/sec

$$(d_c)_{computed} = \left(\frac{m_p V_p \cdot V_o}{4\pi F_{tu}}\right)^{1/3}$$

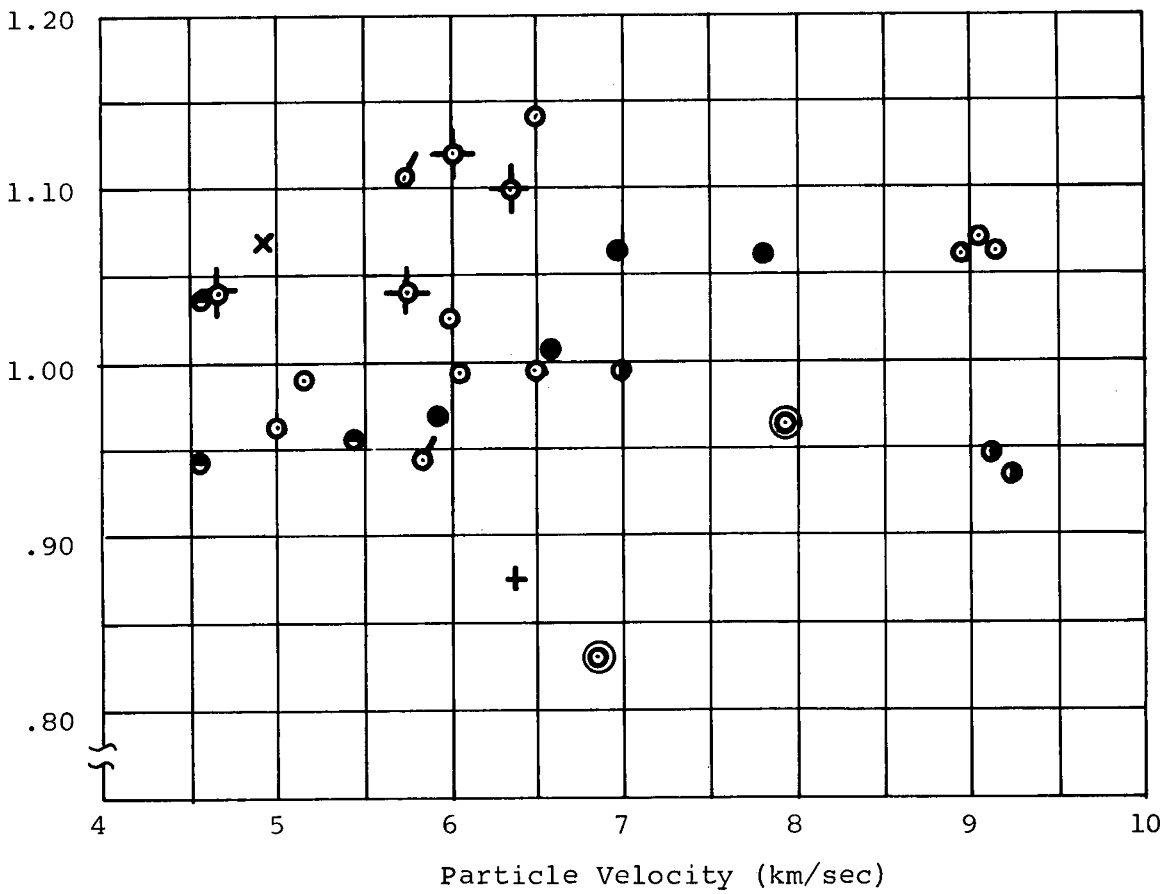

Figure 2. Comparison of Measured and Computed Crater Depths, for Hypervelocity Particle Impact

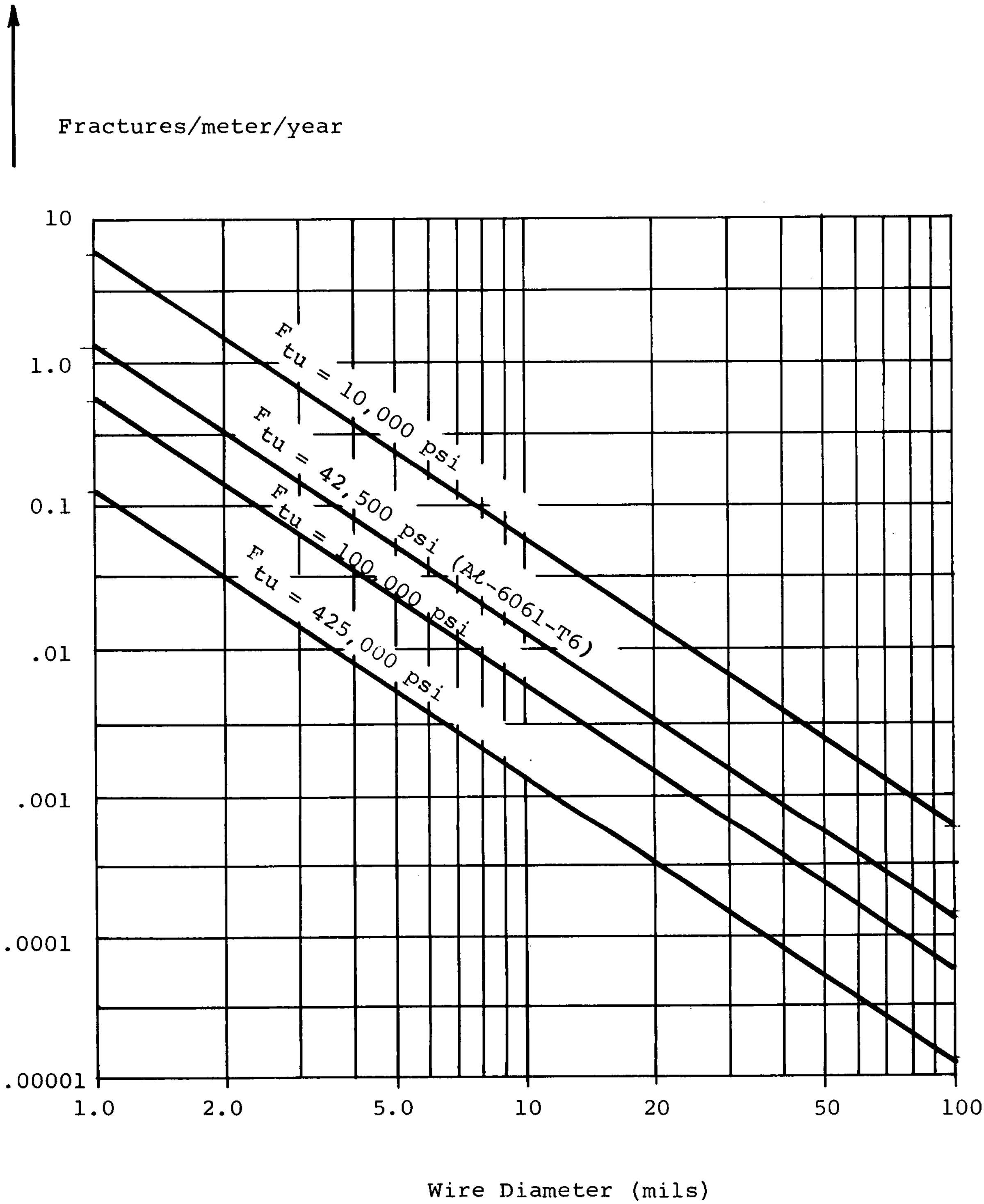

Figure 3. Estimated Fracture Rates For Round Wires

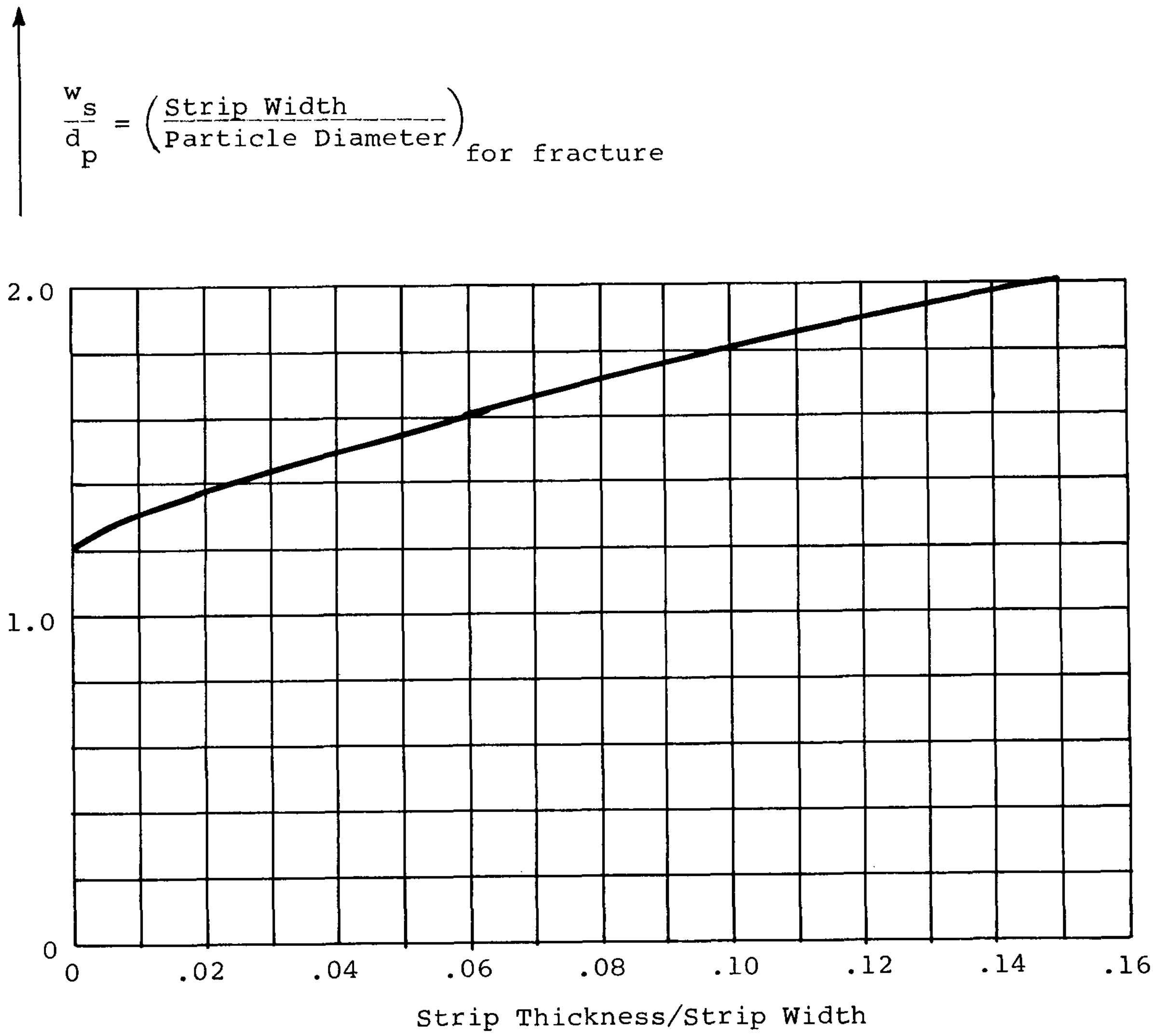

Figure 4. Ratio of Strip Width to Particle Diameter At Fracture vs Strip Geometry

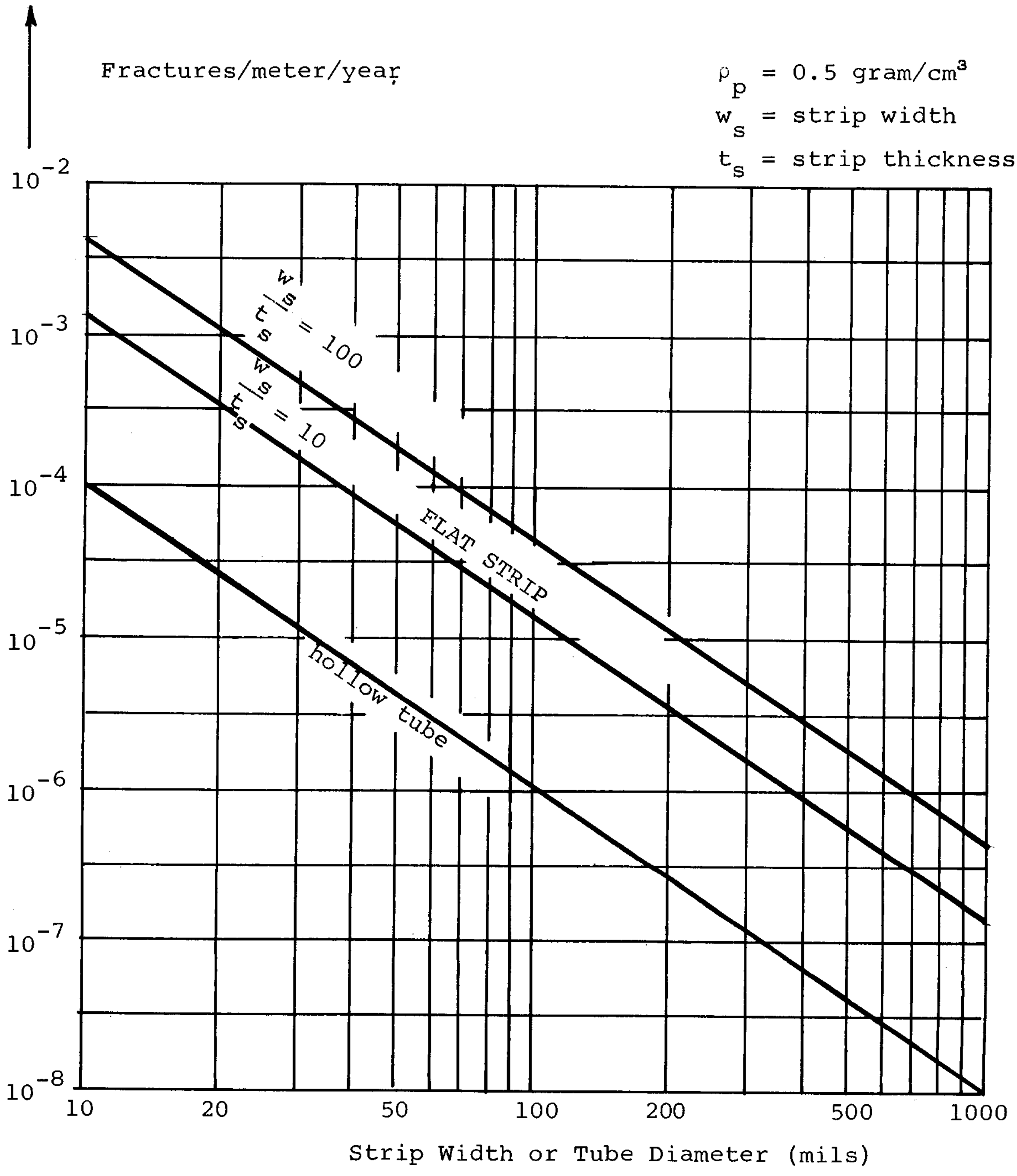

Figure 5. Fracture Rate For Flat Strips and Hollow Tubes

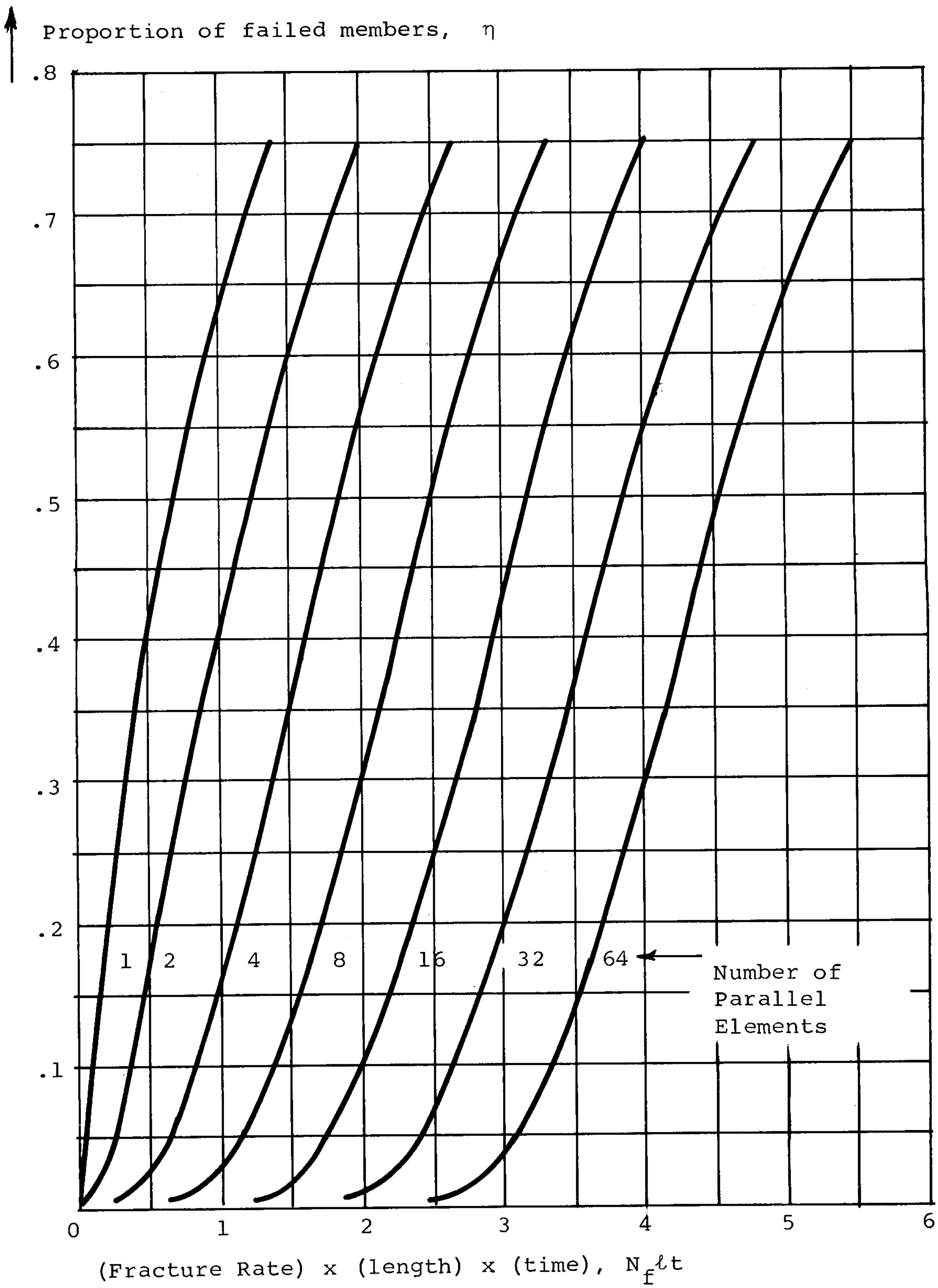

Figure 6. Proportion of Members Failed with Parallel Redundancy

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